Eur. Phys. J. C 8, 507–511 (1999)
DOI 10.1007/s100529901089

THE EUROPEAN PHYSICAL JOURNAL C

Regularized supermembrane theory and static configurations of M-Theory

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Received: 4 September 1998 / Published online: 15 April 1999

Abstract. We suggest that the static configurations of M-theory may be described by the matrix regularization of the supermembrane theory in static regime. We compute the long-range interaction between a M2-brane and an anti-M2-brane in agreement with the 11-dimensional supergravity result.

1 Introduction

The proposed M(atrix) model [1] as a non-perturbative formulation of M-theory [2] has provided a new and effective framework for studying dualities and connections between different string theories [3–7]. This model is the dimensional reduction of 9+1 $U(N)$ SYM theory to 0+1 dimension [8] in the large-N limit, which latter was introduced and studied as the dynamics of N D0-branes [9, 10].

In the initial developments of the supermembrane theory [11,12] in an 11-dimensional supergravity background, it was observed that the existence of κ-symmetry imposes restrictions on the background fields which reduce to the 11-dimensional supergravity field equations. Since M-theory has the 11-dimensional supergravity as its low energy limit, the above observation suggests that every definition of M-theory should be closely connected to supermembrane theory. Thus, M-theory in an infinite momentum frame and supermembrane action in light-cone gauge, written in a matrix form, are related [1].

On the other hand, in the formulation of the M(atrix) model for M-theory the notion of a substructure has played a central role. Therefore it is plausible to expect that the same substructure, in the form of a matrix formulation, should play a role in the framework describing the static configurations of the M-theory.

As there is no definition for covariant M-theory, it is tempting to study it in various gauges: light-cone, static, etc. The above mentioned relations between supermembrane theory and M-theory in light-cone gauge motivates us to search for a similar relation in static gauge. Our starting point is the action of supermembranes in 11 dimensions. By restricting the action to the static part of its phase space, we obtain an action which, after its κ-symmetry is fixed, be written in matrix form.

The resulting matrix action is invariant under $SO(9)$ rotations of target space. Also, the action has a gauge symmetry which corresponds to the world volume area-preserving symmetry. Despite the existence of the gauge symmetry, the interpretation of the model as a dimensional reduction of SYM theory seems impossible.

We introduce solutions for the action, which, as is expected from M-theory, have vanishing quantum corrections. We also calculate the long- range interaction of parallel M2-brane and anti-M2-brane solutions of the matrix model. The result is $W(r) \sim 1/r^6$, which agrees with the uncompactified 11-dimensional supergravity, in direct in contrast to the light-cone M(atrix) theory result in compactified limit $W(r) \sim 1/r^5$.

Conventions and some calculations are gathered in appendices.

2 Static supermembrane action as a matrix model

We use the following notations everywhere:

$$a, b = 0, 1, 2;\ \mu, \nu = 0, 1, ..., 9, 10;$$

$$I, J, K = 1, 2, ..., 9, 10; \quad \text{and } i, j, k = 1, 2, ..., 9.$$

The supermembrane action in 11 dimensions is [13,11]

$$S = \frac{-1}{2} \int \mathrm{d}^3\eta \Big(2\sqrt{-g} + \epsilon^{abc} \bar{\theta} \Gamma_{\mu\nu} \partial_a \theta \times (\Pi_b^\mu \partial_c X^\nu + \frac{1}{3} \bar{\theta} \Gamma^\mu \partial_b \theta \bar{\theta} \Gamma^\nu \partial_c \theta) \Big), \quad (1)$$

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where Π and g are

$$\Pi_a^\mu = \partial_a X^\mu + \bar\theta \Gamma^\mu \partial_a \theta, \qquad g_{ab} = \Pi_a \cdot \Pi_b, \tag{2}$$

and θ is an 11-dimensional Majorana spinor.

The action (1) is invariant under the global supersymmetry (SUSY) transformation

$$\delta X^\mu = -\bar\epsilon \Gamma^\mu \theta, \quad \delta\theta = \epsilon, \tag{3}$$

and also under the local fermionic symmetry, κ-symmetry

$$\delta X^\mu = \bar\kappa (1-\Gamma)\Gamma^\mu \theta, \qquad \delta\theta = (1-\Gamma)\kappa, \tag{4}$$

where

$$\Gamma = \frac{\epsilon^{abc}}{6\sqrt{-g}} {\Pi_a}^\mu {\Pi_b}^\nu {\Pi_c}^\rho \Gamma_{\mu\nu\rho}, \qquad \Gamma^2 = 1.$$

We decompose the coordinates as $\eta_a = (\tau, \sigma_r)$, $r = 1, 2$.

We go to the static regime defined by

$$X^0 \equiv \tau, \qquad \dot X^I \equiv \dot\theta \equiv 0; \tag{5}$$

then the components of g are found to be

$$\begin{aligned} &g_{00} = -1, \qquad -f_r \equiv g_{0r} = -\bar\theta\Gamma^0\partial_r\theta, \\ &g_{rs} = \bar g_{rs} - f_r f_s, \quad \text{and} \quad \bar g_{rs} \equiv \Pi_{rI}\Pi_{sI}; \end{aligned} \tag{6}$$

and it can easily be shown that,

$$\begin{aligned} &g = -\bar g, \\ &\bar g = \det \bar g_{rs} = \frac{1}{2}\epsilon^{rs}\epsilon^{r's'}\bar g_{rr'}\bar g_{ss'} = \frac{1}{2}(\epsilon^{rs}\Pi_r^I \Pi_s^J)^2. \end{aligned} \tag{7}$$

Putting all the above relations into (1), we obtain

$$\begin{aligned} S = \frac{1}{2}\int \mathrm{d}\tau\, \mathrm{d}^2\sigma \bigg(&- e^{-1} - e\bar g \\ &- 2\epsilon^{rs}\bar\theta\Gamma_{0I}\partial_r\theta\partial_s X^I - \epsilon^{rs}\bar\theta\Gamma_{0I}\partial_r\theta\bar\theta\Gamma^I\partial_s\theta \bigg), \end{aligned} \tag{8}$$

where e appears as an auxiliary field for linearizing the action; its equation of motion gives

$$e^2 \bar g \; = 1, \tag{9}$$

which can be used for eliminating e. Due to (9), configurations with $\bar g = 0$ are unacceptable.

The action (1) has a local fermionic symmetry, called κ-symmetry, which allows one to gauge away half of the fermionic degrees of freedom of θ. θ is a 32-component 11-dimensional Majorana spinor and is real in a real representation of Γ matrices, which we use (see Appendix 1). We fix the κ-symmetry by the light-cone gauge [1]: (i.e. $(\Gamma^0 + \Gamma^{10})\theta = \Gamma^+\theta = 0$)

$$\theta = \frac{1}{2}\begin{pmatrix}\lambda \\ \lambda\end{pmatrix}, \qquad \lambda = \lambda^*. \tag{10}$$

[1] In fact, we could do gauge-fixing before restricting the action to its static regime by the ansatz (5).

Then it can be shown that

$$\begin{aligned} &\bar\theta\Gamma_i\partial\theta = 0, \qquad \bar\theta\Gamma_{10}\partial\theta = -\frac{1}{2}\lambda^{\mathrm T}\partial\lambda, \\ &\bar\theta\Gamma_{0i}\partial\theta = -\frac{1}{2}\lambda^{\mathrm T}\gamma_i\partial\lambda, \quad \bar\theta\Gamma_{0,10}\partial\theta = 0. \end{aligned} \tag{11}$$

After integration over τ (which gives $\mathcal{T}$), the action (8) takes the form

$$\begin{aligned} S = -\frac{1}{2}\mathcal{T}\int \mathrm{d}^2\sigma e^{-1}\bigg(&\frac{1}{2}\{X^i, X^j\}^2 \\ &+ (\{X^i, X^{10}\} - \frac{1}{2}\lambda^{\mathrm T}\{X^i, \lambda\})^2 \\ &+ \lambda^{\mathrm T}\gamma_i\{X^i,\lambda\} + 1\bigg), \end{aligned} \tag{12}$$

where

$$\begin{aligned} \{a, b\} &= e\,(\partial_{\sigma_1}a\partial_{\sigma_2}b - \partial_{\sigma_2}a\partial_{\sigma_1}b) \\ &= e\;\epsilon^{rs}\partial_r a\partial_s b, \end{aligned} \tag{13}$$

which satisfies the Jacobi identity.

We can now formulate our matrix model. By the usual substitutions [13,1,14] [2]

$$\{a, b\} \Rightarrow -i\,[a, b], \qquad \int e^{-1}\,\mathrm{d}^2\sigma \Rightarrow \mathrm{Tr}, \tag{14}$$

with the following consequences:

$$\begin{aligned} &\int e^{-1}\,\mathrm{d}^2\sigma\Big(\{a,b\}c\Big) = \int e^{-1}\mathrm{d}^2\sigma\Big(a\{b,c\}\Big) \Rightarrow \\ &\qquad \mathrm{Tr}\;\Big([a,b]c\Big) = \mathrm{Tr}\Big(a[b,c]\Big), \\ &\int e^{-1}\,\mathrm{d}^2\sigma\{a,b\} = 0 \Rightarrow \mathrm{Tr}[a,b] = 0,; \end{aligned} \tag{15}$$

one then finds

$$\begin{aligned} S = -\frac{1}{2}\alpha\mathcal{T}\;\mathrm{Tr}\;\bigg(&\frac{1}{2}\;[X^i, X^j]^2 \\ &+ ([X^i, X^{10}] - \gamma\frac{1}{2}\lambda^{\mathrm T}[X^i,\lambda])^2 \\ &+ i\lambda^{\mathrm T}\gamma_i[X^i,\lambda]\bigg) + \frac{1}{2}\beta\mathcal{T}\;\mathrm{Tr}\;(1). \end{aligned} \tag{16}$$

Here α, β and γ appear due to dimensional considerations in going from the bracket to the commutator and also from integration to trace. We will fix α and β later.

The action (16) has a gauge symmetry which may be identified with area-preserving symmetry of the supermembrane [13]. It is defined by an arbitrary matrix Λ:

$$\begin{aligned} \delta_{\text{gauge}}X^i &= i[X^i, \Lambda], \\ \delta_{\text{gauge}}\lambda &= i[\lambda, \Lambda], \\ \delta_{\text{gauge}}X^{10} &= i[X^{10}, \Lambda]. \end{aligned} \tag{17}$$

[2] There is a factor n for $n\times n$ matrices in going from bracket to commutator and also from integration to trace. Here we absorbed the factor every time in commutator entries.

Furthermore, the action (16) is invariant under SUSY transformations

$$\begin{aligned} \delta X^i &= 0, \\ \delta X^{10} &= \frac{1}{2}\eta^{\mathrm{T}}\lambda, \\ \delta\lambda &= \eta, \end{aligned} \tag{18}$$

where η is an anti-commuting $SO(9)$ spinor, and it can be shown that the above transformations form space-time SUSY algebra

$$\begin{aligned} [\delta_\eta, \delta_{\eta'}]X^i &= 0, \\ [\delta_\eta, \delta_{\eta'}]X^{10} &= \eta'^{\mathrm{T}}\eta, \\ [\delta_\eta, \delta_{\eta'}]\lambda &= 0, \end{aligned} \tag{19}$$

which for X^{10} can be understood as a non-zero translation, because of $\{q_A, q_B\} = \Gamma^{10}P_{10}\delta_{AB}$. Here, the 10th direction is appearing as the 11th direction in the super-Galilean algebra [1,15][3].

3 Solutions with vanishing quantum corrections

In this section we describe certain configurations that are the solutions of the classical equations of motion, and show that the quantum corrections at one-loop order vanish for these configurations. So these solutions, as is expected from similar ones in M-theory, show Bogomol'nyi-Prasad Sommerfield (BPS) behaviour.

The one-loop effective action around the classical solutions

$$X^{10} = \lambda = 0,$$

is computed in Appendix B, and the result is

$$\begin{aligned} W = &\frac{1}{2}\mathrm{Trlog}\left(P_k^2\delta_{IJ} - 2iF_{ij}\right) \\ &- \frac{1}{4}\mathrm{Trlog}\left(P_i^2 + \frac{i}{2}F_{ij}\gamma^{ij}\right) - \mathrm{Trlog}(P_i^2), \end{aligned} \tag{20}$$

assuming the definitions

$$P_i * = [p_i, *], \quad F_{ij} * = [f_{ij}, *], \quad f_{ij} = i[p_i, p_j], \tag{21}$$

where p_i is classical solution of X_i.

Every solution with

$$F_{ij} = 0, \quad \forall i, j, \tag{22}$$

leads to vanishing of the one-loop effective action, due to the following algebra:

$$W \sim (\frac{10}{2} - \frac{16}{4} - 1)\ \mathrm{Trlog}(P_i^2)\ = 0.$$

We next search for these solutions[4].

To begin, we consider a solution of (12) which represents a single flat static membrane. With the conditions $X^{10} = \lambda = 0$, the equations of motion (12) are

$$\{X^i, \{X^i, X^j\}\} = 0.$$

Then

$$X^1 = \sigma_1,\ X^2 = \sigma_2,\ \text{other}\ \ X^i\ =\ 0, \tag{23}$$

represent a single membrane solution,

$$\{X^1, X^2\} = \{\sigma_1, \sigma_2\} = e = 1,$$

due to the equation of motion of e. In the matrix version the conditions $X^{10} = \lambda = 0$ give

$$[X^i, [X^i, X^j]] = 0,$$

which, analogous to (23), leads to

$$X^1 = \frac{L_1}{\sqrt{2\pi n}}q, \quad X^2 = \frac{L_2}{\sqrt{2\pi n}}p, \quad \text{other } X^i = 0, \tag{24}$$

with $[q, p] = i$ and $0 \leq q, p \leq \sqrt{2\pi n}$ eigenvalue distributions. This solution represents a 2-dimensional object extended in X^1 and X^2 directions, and clearly it satisfies (22), thus is stable under quantum fluctuations. Also, because of the spectrum of p and q, the area of the 2-dimensional object (M2-brane) is $L_1 L_2$.

[3] In general, to find the complete SUSY transformations, one must search for those which respect κ-symmetry gauge- fixing by solving the equation

$$\Gamma^+\theta = 0 \leftrightarrow \Gamma^+(\theta + \epsilon + (1 - \Gamma)\kappa) = 0 \Rightarrow \Gamma^+(\epsilon + (1 - \Gamma)\kappa) = 0.$$

This is a constraint equation between SUSY and κ-symmetry parameters ϵ and κ, as global and local spinors, respectively. A rapid solution is $\kappa = 0$ and $\epsilon \sim \begin{pmatrix} \eta \\ \eta \end{pmatrix}$, which leads to SUSY transformations (18). Another closed-form solution seemed inaccessible in our static case. A similar observation is reported as a result of non-linearities of equations of motion [12]. So we just keep (18).

[4] The point-like configurations which may be represented by the solutions

$$X^i = \mathrm{diag}(x_1^i, x_2^i, ..., x_n^i), \quad X^{10} = \lambda = 0,$$

are not acceptable because of vanishing $\bar{g}$ in (9). This is consistent with the fact that the individual 11-dimensional supergravitons, which are candidates for "quark" substructure of our model (due to their role in the infinite- momentum-frame M(atrix) model as "partons") cannot be studied as static configurations in 11 dimensions, because they are massless. This argument will also be supported by the equation of motion of n, the size of the matrices. By inserting the solutions introduced above into the action, one finds

$$S = 0\ + \frac{1}{2}\beta\mathcal{T}n.$$

The equation of motion for n has no solution (it results in $1 = 0$).

There are also solutions corresponding to two parallel M2-branes,

$$
\begin{aligned}
X^1 &= \begin{pmatrix} \frac{L_1}{\sqrt{2\pi n}} q & 0 \\ 0 & \frac{L_1}{\sqrt{2\pi n}} q \end{pmatrix} \equiv p^1, \\
X^2 &= \begin{pmatrix} \frac{L_2}{\sqrt{2\pi n}} p & 0 \\ 0 & \frac{L_2}{\sqrt{2\pi n}} p \end{pmatrix} \equiv p^2, \\
X^3 &= \begin{pmatrix} r/2 & 0 \\ 0 & -r/2 \end{pmatrix} \equiv p^3, \\
\text{other} \quad & X^i = \lambda = 0,
\end{aligned} \tag{25}
$$

extending in X^1 and X^2 directions and at the distance r in the X^3 direction. Again, this solution clearly satisfies (22), which means that the two M2-branes are under the no-force condition.

4 M2-brane and anti-M2-brane long-range interaction

In this section we calculate the long-range interaction between an M2-brane and anti-M2-brane in parallel. Solutions corresponding to two membranes with opposite charges were introduced in [16]:

$$
\begin{aligned}
X^1 &= \begin{pmatrix} \frac{L_1}{\sqrt{2\pi n}} q & 0 \\ 0 & \frac{L_1}{\sqrt{2\pi n}} q \end{pmatrix} \equiv p^1, \\
X^2 &= \begin{pmatrix} \frac{L_2}{\sqrt{2\pi n}} p & 0 \\ 0 & -\frac{L_2}{\sqrt{2\pi n}} p \end{pmatrix} \equiv p^2, \\
X^3 &= \begin{pmatrix} r/2 & 0 \\ 0 & -r/2 \end{pmatrix} \equiv p^3, \\
\text{other} \quad & X^i = \lambda = 0,
\end{aligned} \tag{26}
$$

where $[q,p] = i$. To calculate the potential between these membranes, one must find the one-loop effective action of (16). The one-loop effective action W was introduced in the previous section (and calculated in Appendix 2),

$$
\begin{aligned}
W &= \frac{1}{2}\mathrm{Trlog}\Big(P_i^2\delta_{IJ} - 2iF_{ij}\Big) \\
&- \frac{1}{4}\mathrm{Trlog}\Big(P_i^2 + \frac{i}{2}F_{ij}\gamma^{ij}\Big) - \mathrm{Trlog}(P_i^2),
\end{aligned} \tag{27}
$$

where $P_i * = [p_i, *]$, $F_{ij} * = [f_{ij}, *]$, $f_{ij} = i[p_i, p_j]$.

The calculations of (27) with solutions like (26) are similar to those of [14] for the interaction between two anti-parallel D-strings. For solutions (26) we have $[p_i, f_{ij}] = c - number$, which means that P_i^2 and F_{ij} are simultaneously diagonalizable. Also $[P_1, P_2] \sim i$, which means that P_i^2 behaves like a harmonic oscillator. The steps of calculations are presented in [14], and the result is

$$
W = (-8n)(\frac{L_1 L_2}{2\pi n})^3 \frac{1}{r^6} + O(\frac{1}{r^8}), \tag{28}
$$

which is in agreement with the 11-dimensional supergravity results for the interaction of an M2-brane and anti-M2-brane [16,17]. It is notable that this result is in the uncompactified limit of 11-dimensional supergravity, in contrast to the result of light-cone M(atrix) theory ($W(r) \sim 1/r^5$) [16].

The result (28) can be used for fixing the parameters α and β in (16). By inserting (24) in (16), one finds

$$
S = (\frac{1}{4})\alpha\mathcal{T}(\frac{L_1 L_2}{2\pi n})^2 \, n + \frac{\beta\mathcal{T}}{2}n, \tag{29}
$$

and the equation of motion of n gives

$$
\frac{L_1 L_2}{2\pi n} = \sqrt{\frac{2\beta}{\alpha}}, \tag{30}
$$

resulting in

$$
S = \frac{1}{2\pi}\sqrt{\frac{\alpha\beta}{2}}(\mathcal{T}L_1L_2) = T_M(\mathcal{T}L_1L_2), \tag{31}
$$

in which the second equality is the action of a flat membrane with T_M as its tension. (31) gives

$$
T_M = \frac{1}{2\pi}\sqrt{\frac{\alpha\beta}{2}}. \tag{32}
$$

By comparing (28) with 11-dimensional supergravity interaction [16], one finds

$$
\frac{L_1 L_2}{2\pi n} = \sqrt{\frac{24\pi\mathcal{T}}{T_M}}. \tag{33}
$$

By using (30,32,33) and extracting an irrelevant numerical factor, α and β are fixed as follows:

$$
\alpha = \sqrt{\frac{T_M^3}{\mathcal{T}}}, \quad \beta = 12\pi\sqrt{T_M\mathcal{T}}. \tag{34}
$$

By choosing $\mathcal{T} = T_M^{-1/3}$, the action (16) becomes:

$$
\begin{aligned}
S = -\frac{1}{2}T_M^{4/3}\mathrm{Tr}\Big(&\frac{1}{2}[X^i, X^j]^2 \\
&+ ([X^i, X^{10}] - \gamma\frac{1}{2}\lambda^{\mathrm{T}}[X^i, \lambda])^2 \\
&+ i\lambda^{\mathrm{T}}\gamma_i[X^i, \lambda]\Big) + 6\pi\mathrm{Tr}(1).
\end{aligned} \tag{35}
$$

5 Conclusion and discussions

In this paper we have introduced a matrix model of the static configurations of M-theory. By construction, the large n-limit of the model is, at least classically, equivalent with static supermembrane action after κ-symmetry gauge fixing. We calculated the long-range interaction of a M2-brane and an anti-M2-brane solution in this model and found it to be in agreement with the 11-dimensional supergravity results.

By definition M-theory reduces to various string theories and their compactifications. However, a model for static configurations of M-theory can not be interpreted exactly as a string theory, because there are static configurations in string theories which are not static in uncompactified M-theory (e.g., non-moving D0-branes in IIA theory, which are known to be Kaluza-Klein modes of massless supergravitons of 11-dimensional supergravity, and so they move with the speed of light in 11 dimensions). Notice that the reverse of the above argument is not valid; static configurations in M-theory remain static after compactification. So compactifications of the static matrix model are especially interesting.

Appendix 1 Conventions and notations

Signatures: $g_{ab} = (-,+,+)$,
$\eta_{\mu\nu} = (-,+,+,+,+,+,+,+,+,+,+)$,
$\epsilon^{0rs} = -\epsilon^{rs}$, $\bar{\theta} = \theta^\dagger \Gamma_0$, $[\Gamma^\mu, \Gamma^\nu]_+ = 2\eta^{\mu\nu}$,
$\Gamma^{\mu\dagger} = \Gamma^0\Gamma^\mu\Gamma^0$, $\Gamma^{\mu\nu} = 1/2(\Gamma^\mu\Gamma^\nu - \Gamma^\nu\Gamma^\mu)$,
$\Gamma^0 = \begin{pmatrix} 0 & -1_{16} \\ 1_{16} & 0 \end{pmatrix}$, $\Gamma^{10} = \begin{pmatrix} 1_{16} & 0 \\ 0 & -1_{16} \end{pmatrix}$,
$\Gamma^i = \begin{pmatrix} 0 & \gamma^i_{16} \\ \gamma^i_{16} & 0 \end{pmatrix}$, $\Gamma^+ = \Gamma^0 + \Gamma^{10}$,
${\gamma^i_{16}}^\dagger = {\gamma^i_{16}}^* = \gamma^i_{16}$, $[\gamma^i, \gamma^j]_+ = 2\delta^{ij}$,
$\Gamma^1\Gamma^2 ... \Gamma^9\Gamma^{10} = \Gamma^0$.

Appendix 2 One-loop effective action

The calculation of this part is similar to that in [14]. In this part we decompose the matrices X and θ to classical solutions and quantum fluctuations as follows:

$$\begin{aligned} X^i &= (\ p^i\ )_{\text{class.}} + a^i, \\ \lambda &= (\ 0\ )_{\text{class.}} + \phi, \\ X^{10} &= (\ 0\ )_{\text{class.}} + a^{10}, \end{aligned} \tag{36}$$

where $(...)_{\text{class.}}$ are classical solutions and the remaining right-hand side are quantum fluctuations around classical solutions. After expanding the action (16) up to quadratic terms in fluctuations, and using equations of motion, one finds

$$\begin{aligned} \Delta S = -\mathrm{Tr}\bigg(&\frac{1}{2}\ [p_i, a_J]^2\ +\ [p_i, p_j][a_i, a_j] - \frac{1}{2}[p_i, a_i]^2 \\ &+ \frac{i}{2}\phi^{\mathrm{T}}\gamma^i[p_i, \phi]\bigg). \end{aligned} \tag{37}$$

We have ghosts, because of the gauge invariance introduced in the text,

$$S_{\text{ghost}} = -\mathrm{Tr}\bigg(\frac{1}{2}[p_i, a_i]^2 + [p_i, b][p_i, c]\bigg).$$

By introducing the adjoint operators

$$P_i * = [p_i, *], \quad F_{ij} * = [f_{ij}, *], \quad f_{ij} = i[p_i, p_j], \tag{38}$$

the final form of the action will be

$$S_2 = \mathrm{Tr}\bigg(\frac{1}{2}(a_I P_i^2 \delta_{IJ} a_J - a_i 2iF_{ij}a_j) - \frac{i}{2}\phi^{\mathrm{T}}\gamma^i P_i\phi + bP_i^2 c\bigg).$$

By inserting S_2 into the path integral, the one-loop effective action is obtained:

$$\begin{aligned} W &= -\log \int [\mathrm{d}a][\mathrm{d}\phi][\mathrm{d}c][\mathrm{d}b]\mathrm{e}^{-S_2} \\ &= \frac{1}{2}\ \mathrm{Trlog}\bigg(P_i^2\delta_{IJ} - 2iF_{ij}\bigg) \\ &\quad - \frac{1}{4}\mathrm{Trlog}\bigg(P_i^2 + \frac{i}{2}F_{ij}\gamma^{ij}\bigg) - \mathrm{Trlog}(P_i^2). \end{aligned} \tag{39}$$

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